J. Fluid Mech. (1981), *vol.* 102, *pp.* 431–453
Printed in Great Britain

Unsteady flow in straight alluvial streams. Part 2. Transition from dunes to plane bed

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(Received 27 April 1979 and in revised form 13 February 1980)

The effect of suspended sediment on the modification of individual dunes in unsteady channel flow is analyzed. The model is an extension of that developed by Fredsøe (1979), in which the change of the dune dimensions was analyzed assuming that the transport of sediment mainly occurs as bed load. By including suspended sediment, it is possible to describe dune behaviour at high sediment transport rates, for which the dunes are washed out, so the bed at sufficiently high bed shear stress becomes plane. The model is compared with laboratory experiments and the agreement is reasonable. The effect of temperature variations on dune dimensions is incorporated in the analysis.

1. Introduction

The response of sand wave dimensions (length, height) to changes in hydraulic conditions is of major importance in connection with unsteady flow in alluvial channels. This is because the sandwaves very often contribute significantly to the total hydraulic resistance, due to expansion losses behind the crest of the sandwaves. Because the sandwaves are not able to change their dimensions immediately if the hydraulic conditions change rapidly, the contribution to the flow resistance due to expansion losses is not always determined by the instantaneous hydraulic conditions.

The timescale for the change in the dune height due to a sudden change in the water discharge has been analyzed previously by Fredsøe (1979) applying the assumption that bed-load was the dominant transport mechanism of the sediment. The analysis was based on similarity in the bed shear stress distribution before and after the change in water discharge. The assumption that the sediment is mainly transported as bed-load is valid as long as the effective Shields parameter θ' is small. At low values of θ' it is observed that the ratio between the dune height and the water depth increases with increasing values of θ'. This agrees with the above-mentioned analysis, which predicts that the dunes will always grow in height if the bed shear stress is increased.

At larger values of θ', however, it is observed that the dune height decreases as the bed shear stress increases. At sufficiently large values of θ' this implies that the bed becomes plane, if the critical Froude number has not been exceeded. In case of coarse sand or small water depth the flow may become super-critical at relatively small values of θ'. This leads to the formation of antidunes.

The transition from dunes to plane bed below the critical Froude number has in the

0022-1120/81/4644-3050 $02.00

past been studied using linear stability analysis by Engelund & Fredsøe (1974) and Fredsøe & Engelund (1975). Linear stability theory is only able to predict whether a small disturbance of an originally plane bed will increase with time (forming dunes) or decrease with time (resulting in a plane bed). Hence, changes in an originally dune-covered bed due to changes in the hydraulic conditions cannot be predicted with certainty by use of linear stability theory. Nevertheless, the main results from stability theory agree with observations in nature: below the critical Froude number, the stability theory predicts that bed-load transport causes instability, while the suspended load acts as a stabilizing factor. Hence, the transition between dunes and plane bed depends on the ratio between bed-load and suspended load. This leads to the result that the finer the sediment, the smaller the Froude number at which the transition between dunes and plane bed takes place for given energy slope and water discharge. This is experimentally supported by the data of Guy, Simons & Richardson (1966), as shown by Engelund & Fredsøe (1974, figure 5.3).

The purpose of the present paper is to investigate whether the height of dunes on an originally dune-covered bed will increase or decrease as the hydraulic conditions are being changed. Further, the time scale for changes in the dune dimensions is calculated in order to investigate unsteady flow in alluvial rivers. The present model may be regarded as an extension of the model by Fredsøe (1979), in which the suspended sediment is now included in order to describe the unsteady behaviour at high sediment transport rates as suggested by the results from linear stability theory. The present analysis is simplified to treat regular two-dimensional, nearly triangular dunes, as was done in the above mentioned paper.

2. Theory for a sudden change in bed shear stress

Let us consider a dune-covered bed given by

$$h = h(x-a_1 t), \tag{1}$$

where h is the bed elevation, x a co-ordinate in the flow direction, t time and a_1 the migration velocity. In the equilibrium case, the dunes travel downstream with the velocity a_1 without changing the form given by (1). The continuity equation of sediment is

$$\partial q_1/\partial x = -(1-n)\,\partial h/\partial t, \tag{2}$$

where q_1 is the local sediment discharge and n the porosity. q_1 is the sum of bed load q_{b1} and suspended load q_{s1}:

$$q_1 = q_{b1}+q_{s1}. \tag{3}$$

For a bed in equilibrium, (1) and (2) yield

$$\partial q_1/\partial x = (1-n)\,a_1\,\partial h/\partial x. \tag{4}$$

The migration velocity a_1 is determined by the amount of sediment which is deposited on the front just downstream of the crest of the dunes. The slope of this front is nearly equal to the angle of repose of the bed material. Just behind the crest, a separation zone is present in the lee of the dune as sketched in figure 1. Between this separation zone and the main flow an intermediate zone is present, which contains large turbulent fluctuations, as can be seen from the measurements by Raudkivi

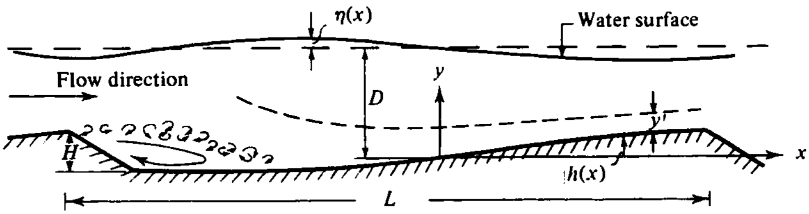

FIGURE 1. Definition sketch.

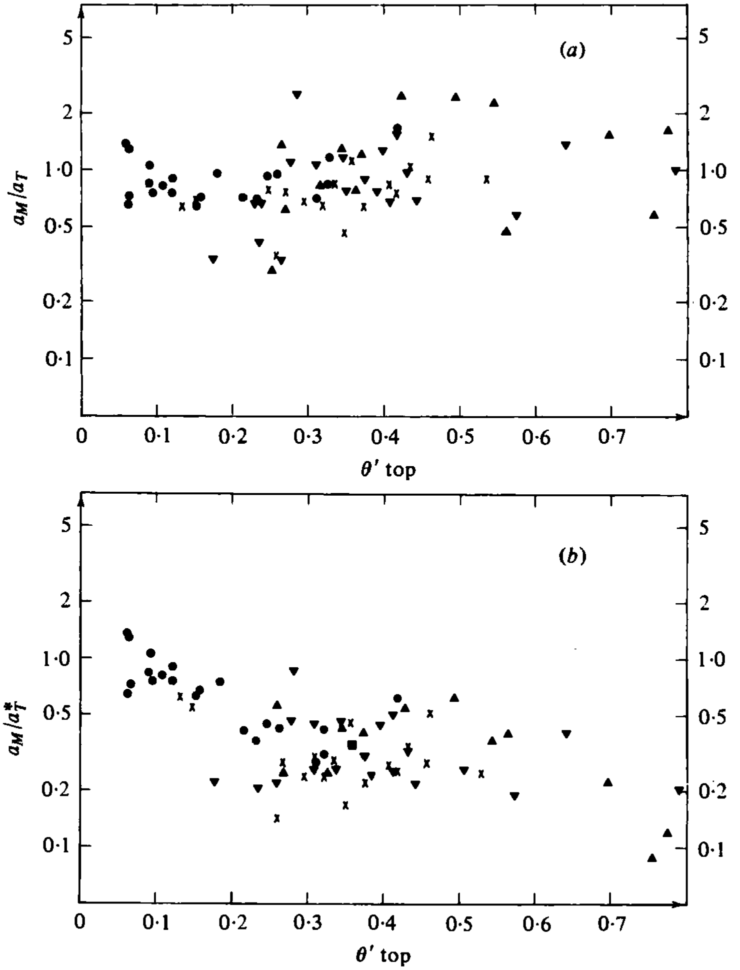

FIGURE 2. Comparison of calculated and measured dune migration velocity. ▲, 0·19 mm; ▼, 0·27, 0·28 mm; ×, 0·32, 0·33 mm; ●, 0·93 mm. (The experimental data are from Guy *et al.* (1966).)

(1963). In the main flow also, the turbulent intensity is increased just downstream of the crest, due to the sudden increase in the depth.

Because of this increased turbulent intensity only a very small fraction of the sediment carried in suspension by the water above the crest of the dune must be expected to be trapped in the separation zone and hence settle close to or on the front of the dune. On the other hand, it is natural to assume that the major part of those grains which are rolling or jumping on the dune crest will be deposited on the dune front, because the bed shear stress is significantly reduced in the separation zone. In a previous paper, Engelund & Fredsøe (1976) define the bed-load as the particles in the lowest layer of moving grains just above the immobile bed. By use of this definition of bed-load, it is now assumed that only that part of the total load that is transported as bed-load at the dune crest will be deposited on the dune front. Then the migration velocity a_1 of the dune is determined by

$$a_1 = \left(\frac{q_{b1}}{(1-n)\,H}\right)_{\text{top}}, \tag{5}$$

where H is the total height of the dune, as shown in figure 1. It is possible to test the validity of the above stated assumptions by comparing measured and calculated migration velocities of dunes. The result of such a comparison is depicted in figure 2. The data are taken from the comprehensive data material by Guy *et al.* (1966). All data denoted as dunes or transition are incorporated in the analysis. In figure 2*a*, the theoretical value of the migration velocity a_T is calculated by use of (5). The bed-load transport is calculated by use of the bed-load formulae given by Engelund & Fredsøe (1976), in which the local value of the effective Shields parameter θ' is based on the local value of the mean flow velocity just above the dune crest. As seen in figure 2*a*, the ratio between the measured dune-migration velocity a_M and the calculated one a_T is close to unity and independent of θ' (and hence the amount of suspended sediment), which confirms the validity of (5). In figure 2*b* a similar comparison is depicted, in which the theoretical migration velocity a_T^* is determined from the assumption that the total amount of sediment will be deposited on the dune front. This total transport is also determined by the method suggested by Engelund & Fredsøe (1976). The suspended sediment is calculated under the assumption that it is determined by the local equilibrium conditions as illustrated below. It is seen that this yields a significant over-estimate of the migration velocities at high values of θ', so from this figure it can be concluded that only an insignificant part of the suspended load settles on the dune front. A large part of the scatter in figure 2 originates from the deviation between calculated and actual load.

If the bed shear stress is now changed suddenly over the bed given by (1), the rate of local bed-load transport will change to q_{b2}, while the suspended load is given by q_{s2}. The total load is called q_2.

The initial change in the dune bed height H is found by the expression

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + a_2\frac{\partial H}{\partial x}, \tag{6}$$

where a_2 is determined in a manner analogous to (5).

$$a_2 = \left(\frac{q_{b2}}{(1-n)\,H}\right)_{\text{top}}. \tag{7}$$

Also the local change in bed elevation with time takes a form similar to (2):

$$\frac{\partial h}{\partial t} = -\frac{1}{(1-n)}\frac{\partial q_2}{\partial x}. \tag{8}$$

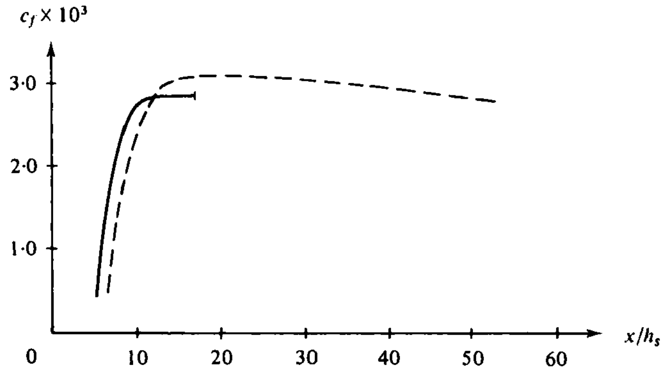

FIGURE 3. Variation in c_f after a rear-facing step. ---, zero pressure gradient (Bradshaw & Wong 1972); ——, favourable pressure gradient (Raudkivi 1963).

By inserting (4), (5), (7) and (8) into (6), the following expression for the initial change in dune height, just after the water discharge has been changed, is obtained

$$\frac{dH}{dt} = \frac{1}{(1-n)}\left\{\frac{q_{b2}}{q_{b1}}\frac{\partial q_1}{\partial x} - \frac{\partial q_2}{\partial x}\right\}_{\text{top}}. \tag{9}$$

As seen from (9), knowledge about the longitudinal variation in the rate of sediment transport is needed in order to calculate the change in the dune height. Two problems arise in this connection: (i) in order to calculate the rate of bed-load, the variation in θ' must be known, because the local transport of bed-load is approximately a function of the local value of θ' only. (ii) the local rate of suspended sediment may be difficult to estimate. This is because it takes some time for the suspended sediment to react on spatial changes in the hydraulic conditions due to the finite settling velocity of the grains in suspension. The two above-mentioned items are considered in the next two sections.

3. Bed shear stress distribution along a dune

The flow picture just behind the dune top is characterized by a high turbulent intensity, and is rather difficult to describe. In the beginning, the flow picture is very much like that for the flow down a rearward-facing step. This flow has been studied rather intensively by, for instance, Bradshaw & Wong (1972), who measured the flow picture along a distance equal to 52 times the stepheight h_s, where h_s was 2·5 cm and the depth was 12·5 cm. They found that the disturbance from the step was damped rather slowly. The dotted curve in figure 3 shows the measured variation in the factor c_f defined by $c_f \equiv (U_f/V)^2$, where V is the mean velocity and U_f the bed shear velocity. It is seen that c_f even at $x = 50\,h_s$ has not obtained the equilibrium value. However, the variations in c_f are rather weak if x exceeds 15–20 h_s, so the friction factor in fact may be taken to be approximately a constant in this region.

In the case of a dune-covered bed, the flow will converge after the step due to the presence of the next dune, so the disturbances from the step will be damped more rapidly compared with a flow with zero pressure gradient. To the author's knowledge, flow with favourable pressure gradients after a rearward-facing step has not been examined extensively. Raudkivi (1963) measured the bed shear stress distribution along a fixed ripple surface for which the length of the ripple was 17 times the height. The ripple height was 2·23 cm. From his experiments, c_f based on the local mean velocity can be calculated; and is depicted in figure 3 by the solid line. It is seen that it becomes constant a little faster than in the flow with a zero pressure gradient. From measurements over a fixed plane bed, Raudkivi measured a c_f value equal to 0·00290 in the equilibrium state, so a drop in c_f at larger values of x must not be expected to take place in a convergent flow.

The ratio of wavelength to waveheight in the case of dunes is normally about 30–100, which is somewhat larger than for ripples. Hence, it must be expected that the friction factor can be taken as constant on the major part of the dune surface. This is in accordance with the point of view of Engelund & Hansen (1972), who suggested that a turbulent boundary layer of the thickness y' will develop along the dune surface as sketched in figure 1. They calculated the friction factor from the expression

$$V/U_f' = \sqrt{2/f'} = 6+2{\cdot}5\ln(y'/k), \tag{10}$$

where U_f' is the effective friction velocity along the dune surface, and f' the corresponding friction factor due to pure skin friction. k is the bed roughness. An expression very similar to (10) was originally suggested by Einstein (1950) from a different point of view.

An additional test of the above statement that the friction factor can be taken as constant on large parts of the upper dune surface can be obtained by use of the continuity equation for sediment, as long as bed-load is the dominating transport form. If (10) is valid, the variation in θ' along the dune is given by

$$\theta' = \theta_{\text{top}}(1-H/2D)^2/(1-h/D)^2. \tag{11}$$

In case of pure bed load (4) can be written as

$$q_{b1} = \left(\frac{q_{b1}}{H}\right)_{\text{top}}(h+\tfrac{1}{2}H), \tag{12}$$

in which the migration velocity has been calculated from (5). The bed load is assumed to vanish in the trough, where the bed shear stress vanishes.

From (12), the local effective bed shear stress can be found as a function of the bed level, because the non-dimensional bed load is a function of the non-dimensional bed shear stress only, as argued by Fredsøe (1979)

$$\Phi_{b1} = \Phi_{b1}(\theta'), \quad \Phi_{b1} = q_{b1}[(s-1)gd^3]^{-\frac{1}{2}}. \tag{13}$$

Inserting (13) into (11) yields by differentiation

$$\frac{\partial \Phi_{b1}}{\partial x} = \frac{d\Phi_{b1}}{d\theta'}\frac{2\theta'}{(1-h/D)}\frac{\partial(h/D)}{\partial x}, \tag{14}$$

while (12) differentiated is

$$\frac{\partial \Phi_{b1}}{\partial x} = \frac{\partial(h/D)}{\partial x}\left[\Phi_{b1}\frac{D}{H}\right]_{\text{top}}. \tag{15}$$

Eliminating $\partial(h/D)/\partial x$ from (14) and (15) we obtain

$$\frac{H}{D}\bigg/\left(1-\frac{1}{2}\frac{H}{D}\right) = \left[\frac{\Phi_{b1}}{2\theta\, d\Phi_{b1}/d\theta}\right]_{\text{top}}. \tag{16}$$

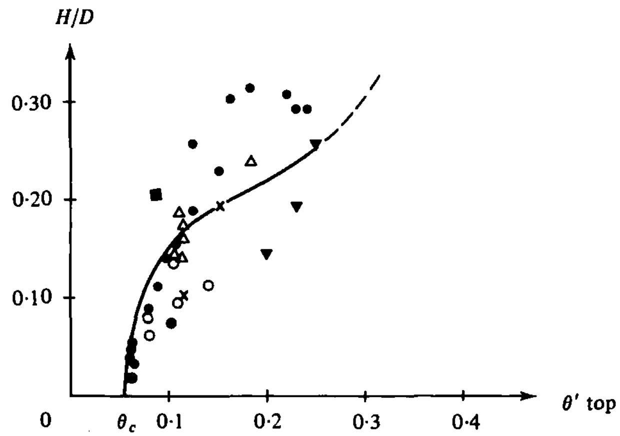

FIGURE 4. Variation in H/D with θ'. ——, equation (16). The symbols for the data are as in figure 2. Further: $\triangle$, 0·30 mm; $\bigcirc$, 1·00 mm. (Data by Gee (1973).)

This relation is depicted in figure 4. The right-hand side is evaluated by use of the bed load transport formulae by Engelund & Fredsøe (1976). The data from Guy *et al.* (1961) are shown in figure 4 together with the equilibrium data measured by Gee (1973). These last data are used later in the paper in connection with unsteady flow. Only data where $\theta' < 0{\cdot}25$ are included in figure 4 in order to avoid significant amounts of suspended load, which makes (10) invalid. The agreement between (15) and the data is reasonable and supports the supposition that the disturbances in the flow picture from the former dune crest are damped out before the next dune top.

4. Behaviour of suspended load along a dune surface

The vertical concentration profile of suspended sediment is, like the velocity profile, rather complex and difficult to describe just downstream of the dune front. For this reason, the length scale required for disturbances in the concentration profile to be damped out in a converging flow is now investigated by an approximate method. In this fashion, the longitudinal variation in the suspended load close to the dune top can be calculated.

The distribution of suspended sediment in a vertical is found from the continuity equation for suspended sediment:

$$dc/dt = w\,\partial c/\partial y + \epsilon \nabla^2 c, \tag{17}$$

in which c is the volumetric concentration of suspended sediment, w the full-velocity, y the vertical co-ordinate with origin in the mean bed level and ϵ the eddy viscosity.

Two approximations are introduced in order to treat the problem analytically:

(i) The eddy viscosity is assumed to be constant over the depth. As a consequence, a finite slip velocity U_b must be introduced at the bed as outlined by Engelund (1964, 1970). This slip-velocity model describes the flow in convergent flow very well, as demonstrated by Engelund (1964). Furthermore, a nominal bed concentration c_{b0} of the suspended sediment must be introduced. For a uniform flow, this implies the following solution to (17):

$$c = c_{b0} \exp(-wy/\epsilon). \tag{18}$$

(ii) As in (18), the distribution of suspended sediment in the non-uniform flow is still assumed to be exponential, but the steepness is variable

$$c = c_{b0} \exp[-\alpha(y-h)], \quad \alpha = \alpha(x), \tag{19}$$

where c_{b0} is the local nominal bed concentration. The variation in the local bed concentration can be found from the variation in the real bed concentration c_b by the relation obtained by Fredsøe (1978)

$$dc_b/c_b = dc_{b0}/c_{b0}, \tag{20}$$

where c_b is a function of the local dimensionless bed shear stress θ' (Engelund & Fredsøe 1976).

Neglecting the slow migration velocity of the dunes compared with the flow velocity, (17) can be written as

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = w\frac{\partial c}{\partial y} + \epsilon\nabla^2 c, \tag{21}$$

where u and v are the velocity components in the x and y directions. As most of the suspended sediment is located close to the bed, u can be replaced by the local slip velocity U_b, which is a good approximation since the variation in the slip-velocity profile over the depth is weak. Analogously, the vertical velocity is written as

$$v = U_b\, \partial h/\partial x, \tag{22}$$

owing to the kinematic boundary condition for the flow at the bed. Finally, the operator ∇^2 is replaced by $\partial^2/\partial y^2$, since variations in suspended concentrations in case of sand are much larger in the vertical than in the horizontal direction.

Inserting (19) into (21), we now obtain, by integration over a vertical,

$$U_b \frac{d}{dx}\{c_{b0}\alpha^{-1}(1-e^{-\alpha(D-h)})\} = (1-e^{-\alpha(D-h)})(\alpha\epsilon - w)\, c_{b0}. \tag{23}$$

Since (23) is an integrated version of (17), the small errors introduced by (19) are reduced. However, (19) does not fulfil the boundary condition that the vertical sediment flux vanishes at the water surface; it yields the following upward-directed flux at the surface

$$-\left[\epsilon\frac{\partial c}{\partial y} + wc\right] = (\alpha\epsilon - w)\, e^{-\alpha(D-h)} c_{b0}.$$

This quantity must be added to the right-hand side of (23), which represents the depth-integrated continuity equation of sediment; the corrected version then reads

$$U_b \frac{d}{dx}\{c_{b0}\alpha^{-1}(1-e^{-\alpha(D-h)})\} = (\alpha\epsilon - w)\, c_{b0}. \tag{24}$$

This small correction is of course only important if the surface concentration of suspended sediment is significant, as it normally will not be.

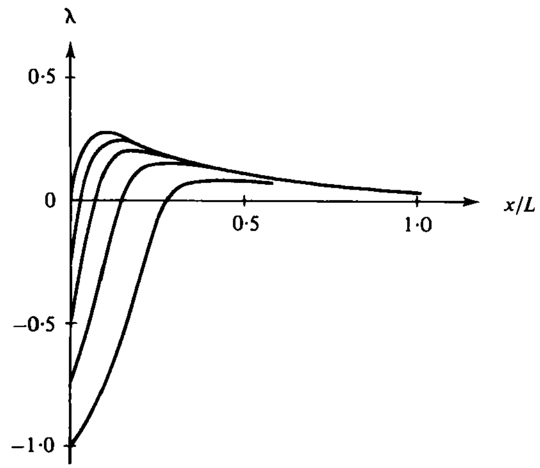

FIGURE 5. Variation in λ along the dune surface.

(24) is a simple first order differential equation in α, which can easily be solved numerically. Introducing λ by the definition

$$\alpha = w(1+\lambda)/\epsilon, \tag{25}$$

λ becomes a measure of the deviation in the suspended profile from the local equilibrium profile given by (18).

In figure 5 the variation in λ along the dune surface obtained from the analysis above is given for flow over a single dune of triangular form. The bed shear stress is assumed to vary as given by (11), which is a good approximation on a large part of the dune as demonstrated in figure 4. The data used to obtain figure 5 are taken from the transition between run 19R and run 21 in the experiments by Gee (1973), just after the water discharge has been increased in run 19R.

The solution of λ from (24) depends on the value of λ_0 in the upstream part of the dune. If no dunes are present in the bed until the one considered, λ_0 equals zero at this place. This yields a solution such that at the top λ approaches a value equal 0·03, so the suspended load is a little smaller than the equilibrium load. However, it cannot be expected that the suspended load is in local equilibrium as it enters a new dune, because of the disturbance in the concentration profile just after the former dune top. For this purpose, the variation in λ with different values of λ_0 is shown in figure 5. It is seen that λ approaches the same value on the upper part of the dune even if the load is uniform distributed over a vertical ($\lambda_0 = -1$). In this case the amount of suspended load is highly exaggerated, since the concentration over the whole depth has been put equal to the bed concentration.

λ will always tend to a nearly constant value at the upper part of the dune, which can be demonstrated by the linearized version of (24). As the bed shear stress is proportional to the slip velocity squared, we obtain

$$d\theta'/\theta' = 2\,dU_b/U_b = 2\,dh/D.$$

The dune height is assumed to be small relative to the waterdepth. If λ is also taken to be small and the dune is triangular, (24) reads

$$d\lambda/dx + \kappa\lambda = \gamma, \tag{26}$$

in which

$$\kappa = w^2/\epsilon U_b \quad \text{and} \quad \gamma = 2\frac{\theta'}{c_{b0}}\frac{dc_{b0}}{d\theta'}\frac{H}{DL}. \tag{27}$$

The solution to (26) is

$$\lambda = \lambda_0 \exp(-\kappa x) + \frac{\gamma}{\kappa}[1 - \exp(-\kappa x)], \tag{28}$$

so

$$\lambda_{\text{top}} \simeq \frac{\gamma}{\kappa}, \tag{29}$$

if $\exp(-\kappa L) \ll 1$. In the slip-velocity mode, ϵ is determined by (Engelund 1970)

$$\epsilon = 0{\cdot}077\, U'_f y'. \tag{30}$$

A rough estimate of κ can be obtained as follows: the condition for a particle to go in suspension is that w/U'_f is about unity, see for instance Sumer (1978). This requirement will most often lead to a mean value of w/U'_f about 0·5–0·7 for sand. This is for instance the range for the data by Guy *et al.* (1966). The length of dunes is 6–15 times the water-depth, and the thickness of the boundary layer y' is about half the water depth.

Finally, U_b/U'_f is related to the friction factor by

$$U_b/U'_f = (V/U'_f) - 4{\cdot}3, \quad V/U'_f = \sqrt{2/f'},$$

which can easily be obtained from Engelund's slip-velocity model (1964). Hence, U_b/U_f is about 10–15. This yields values of κL of order 5–25 for dunes, so (29) will normally be valid. However, this may be checked for individual runs, as the scatter in w/U'_f is large.

The local transport of suspended sediment q_s is found by

$$q_s = \int_h^D cU\,dy \simeq U_b\frac{\epsilon}{w}c_{b0}/(1+\lambda) = q_{se}/(1+\lambda), \tag{31}$$

where q_{se} is the suspended load due to local equilibrium. As seen from (28), the variation in λ is very weak close to the dune top, so the longitudinal variation in the suspended sediment transport is given by

$$\frac{dq_s}{dx} = \left(\frac{1}{1+\lambda}\frac{dq_{se}}{dx}\right)_{\text{top}}, \tag{32}$$

where λ_{top} is determined by (29). λ_{top} is normally a small quantity: by use of (20), (27) and (28) λ_{top} is given by

$$\lambda_{\text{top}} = 2\frac{\theta'}{c_b}\frac{dc_b}{d\theta'}\frac{H}{D}\bigg/\kappa L,$$

where

$$\frac{\theta}{c_b}\frac{dc_b}{d\theta} \approx 0{\cdot}15/\theta^2$$

which can be obtained from the formulae for c_b given by Engelund & Fredsøe (1976).

Run	F	q (m²/s)	Mean θ'	H (cm)	L (m)	D (cm)	Duration of transition T (min) measured	theory	C
19R	0·34	0·035	0·100	1·4	0·77	10·2	—	—	—
19R → 21	0·57	0·068	0·29	1·4	0·77	11·4	15	15	1·2
21	1·04	0·068	0·54	0·55	1·05	7·6	—	—	—
21 → 22	0·53	0·035	0·16	0·55	1·05	7·6	45	61	2·7
22	0·33	0·035	0·097	1·35	0·70	10·4	—	—	—
25	1·03	0·074	0·57	0·60	1·45	8·1	—	—	—
25 → 26	0·43	0·035	0·13	0·60	1·45	8·7	93	165	2·1
26	0·325	0·035	0·095	1·60	0·97	10·5	—	—	—
27	0·56	0·017	0·14	1·10	0·95	4·5	—	—	—
27 → 28	0·82	0·074	0·45	1·10	0·95	9·4	13	6	1·5
28	1·06	0·024	0·59	Plane bed		7·9	—	—	—
33	0·36	0·023	0·093	1·35	0·77	7·4	—	—	—
33 → 34	0·64	0·074	0·33	1·35	0·77	11·0	19	17	3·3
34	1·11	0·074	0·63	Plane bed		7·7	—	—	—

TABLE 1.

As the dune height is of order 0·1–0·2 of the waterdepth, λ_{top} is in the interval

$$10^{-3}/\theta^2 < \lambda_{top} < 10^{-2}/\theta^2.$$

The amount of suspended load is negligible if $\theta' < 0{\cdot}20$, so normally λ will be small.

5. Comparison with laboratory experiments

From (9) and (32), the change in dune height due to a sudden change in bed shear stress can be calculated. In this section, a comparison with Gee's (1973) fine grain experiments ($d = 0{\cdot}30$ mm) is carried out, similar to the comparison with Gee's coarse grain experiments ($d = 1{\cdot}00$ mm) in the previous paper by Fredsøe (1979). In these latter experiments, no suspended sediment was present at all. Gee obtained a change in the bed shear stress by changing the water discharge suddenly. In three experiments, the water discharge was increased, and in two cases the water discharge was decreased. The most important data and evaluated parameters are arranged in table 1.

Because of the small water depth, the Froude number is rather high in some of Gee's experiments. This implies that the water surface in subcritical flow must undulate in 90° out of phase with the bed, as sketched in figure 1. As long as the dune length is several times the water depth, the ratio between the surface elevation η and the bed elevation h is very close to

$$\eta/h = F^2/(F^2-1), \tag{33}$$

if h is small compared with the water depth. This expression has been experimentally verified by Binnie & Williams (1966) for Froude numbers smaller than about 0·9.

Hence, the longitudinal variation in the bed shear stress is given by

$$\left(\frac{d\theta'}{dx}\right)_{top} = \left(\frac{2\theta'}{(1-H/[(1-F^2)\,2D])D}\,\frac{1}{1-F^2}\,\frac{dh}{dx}\right)_{top} \simeq \frac{2\theta'_{top}}{(1-H/[(1-F^2)\,2D])}\,\frac{H}{(1-F^2)\,DL}, \tag{34}$$

which is obtained from (11) by modifying the local depth using (33). (34) fails if the Froude number is close to critical, where dissipative terms make (33) invalid. Hence in runs 21 and 25, $d\theta/dx$ must be estimated in another way. In runs 21 and 25, the flow is still sub-critical as the critical number is somewhat larger than unity in rotational flow (Fredsøe 1974).

In the case of a dune-covered bed in *equilibrium*, the longitudinal bed-shear variation can alternatively be found as follows. The longitudinal variation in sediment transport can be written as

$$\frac{dq}{dx} = \left(\frac{dq_b}{d\theta'} + \frac{1}{1+\lambda}\frac{dq_{se}}{d\theta'}\right)\frac{d\theta'}{dx}, \tag{35}$$

obtained from (13) and (32). By combining (4) and (5), a related expression is obtained:

$$\frac{dq}{dx} = \frac{q_{b,\,\mathrm{top}}}{H}\frac{dh}{dx} \simeq \frac{q_{b,\,\mathrm{top}}}{L}. \tag{36}$$

for a nearly triangular dune. (35) and (36) result in the following expression for the longitudinal bed-shear variation for an equilibrium bed:

$$\left(\frac{dx}{d\theta'}\right)_{\mathrm{top}} = \left[q_b\bigg/\left(L\left(\frac{dq_b}{d\theta'} + \frac{1}{1+\lambda}\frac{dq_s}{d\theta'}\right)\right)\right]_{\mathrm{top}}. \tag{37}$$

The advantage of (37) compared with (34) is that in (37) only knowledge about the absolute value of θ' is needed, while (34) requires knowledge about the gradient of θ'. This is very sensitive to the undulations of the water surface, while the absolute value of θ' at the top does not vary much as long as η is small compared with the total water depth.

Hence, in runs 21 and 25, which are equilibrium runs, (37) is used to evaluate $d\theta/dx$, while (34) has been used in all other runs. In runs 21 and 25, the bed undulations are very small, so η is small compared with the depth.

The initial change in dune height with time is calculated from (9). Gee did not measure the change in dune height directly, but measured the variation in water depth with time, which is related to the dune height for given water discharge, energy gradient and dune length. The procedure for data analysis described in Fredsøe (1979) is also used in the present article.

Gee carried out six experiments with fine sand. In four of these (the first four in table 1), the slope of the water surface S_e was kept constant during the run and parallel to the mean bed. In the two last runs, the slope was changed, as Gee tried to keep the bed shear portion of energy slope S_b fixed throughout the transition. This was done by estimating the wall friction factor f_w to be 0·00625. The width B of the flume was 2 feet. The energy slope S_w due to wall shear from the two walls is

$$S_w = 2f_w q^2/2gBD^2, \tag{38}$$

where D is the water depth and q the specific discharge. As this slope is changed during the transition, S_b should be changed proportionally according to Gee's idea. By doing so, a change in the dune height (for instance a decrease) implies that the coherent decrease in water depth will be exaggerated: because S_w will increase as seen from (38), S_b and S_e must be increased. In run 29 → 30 (Gee's notation) the initial water slope is increased by a factor 2·5, while the ratio S_w/S_b is increased from 0·24 to 0·42.

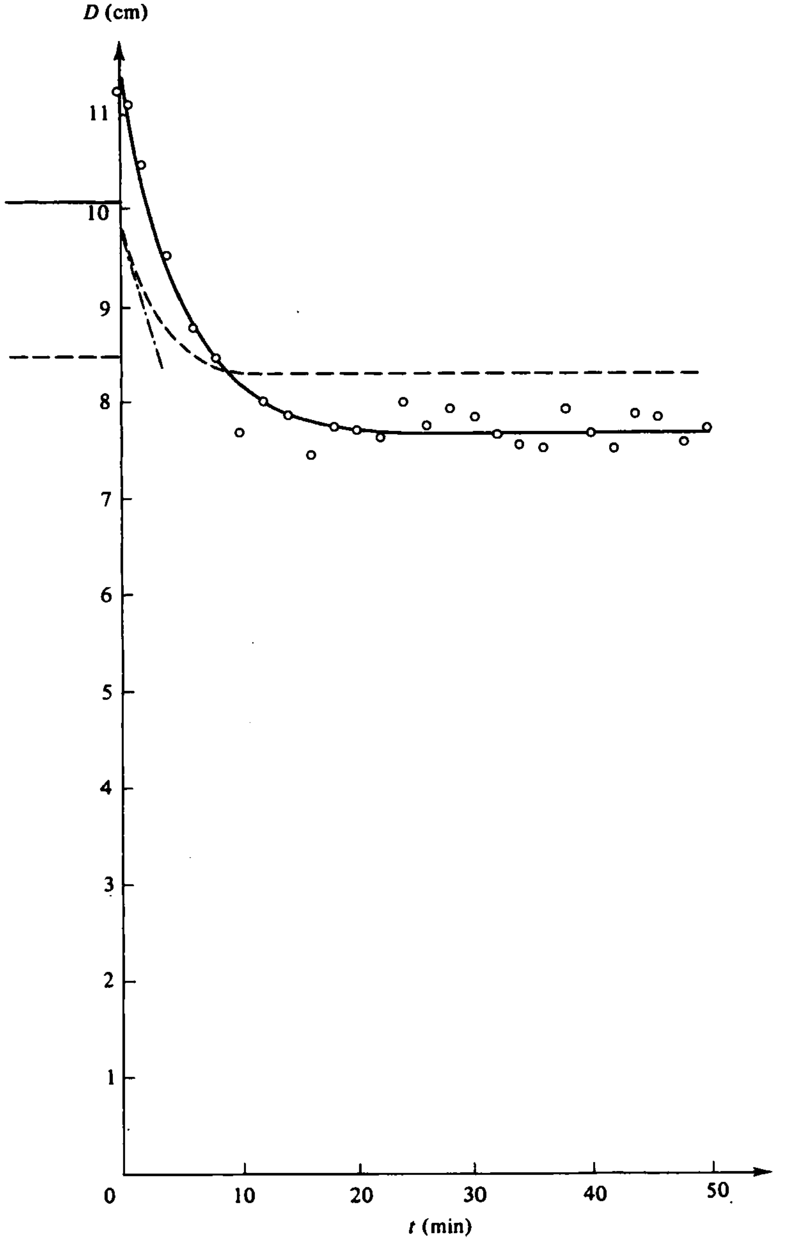

FIGURE 6 (*a*). For legend see p. 447.

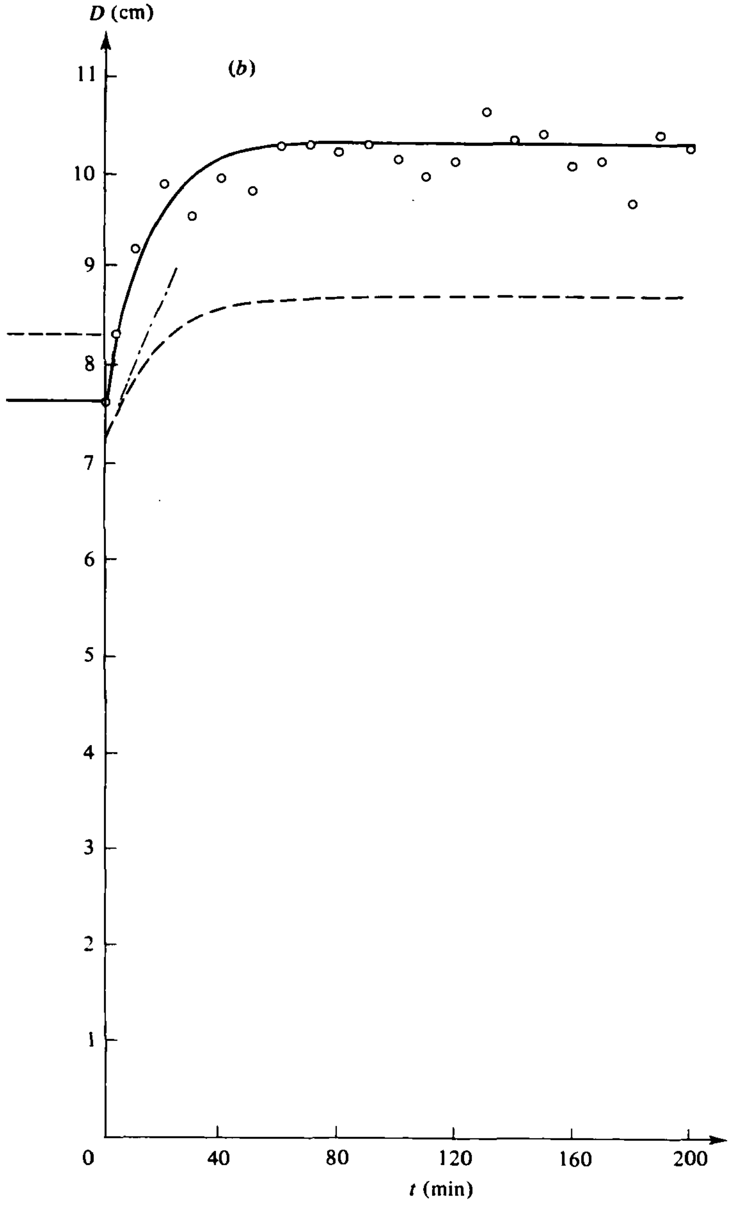

FIGURE 6(b). For legend see page 447.

The change in S_e is too large in this introductory experiment with variable surface slope. In fact, the measured variation with time in water depth agrees very well with calculations, in which the bed friction factor f_b is taken as a constant, so the dimensions of the dunes are kept constant. However, the variations in depth due to changes in dune height is much smaller than those introduced by the changes in the energy slope. Hence, although the theoretical findings fit Gee's measurement rather well in run 29 $\rightarrow$ 30, this experiment is omitted in the comparison, as nothing can be evaluated concerning the time scale for changes in dune height.

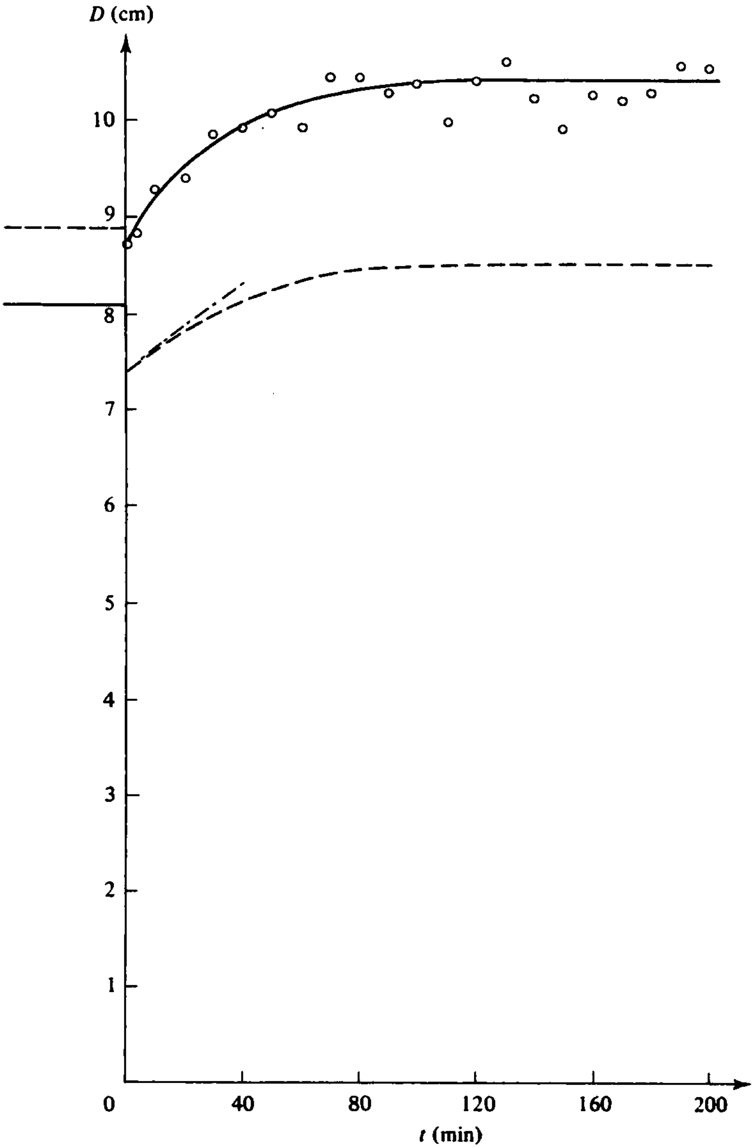

FIGURE 6 (c). For legend see page 447.

A similar problem arises in run 33 → 34, where the slope changes by a factor 1·68 during the transition (after the water discharge has been increased) while S_w/S_b increases from 0·23 to 0·29. However, this implies a more moderate exaggeration of the increase in S_e, and the change in dune height causes a change in the water depth of same order as does the change in S_e. If S_e is changed with fixed dune height, the water depth will be 0·914 times the original water depth D_0 after 5 min, while the corresponding value is 0·75 if variations in S_e and the dune height are both taken into account.

In figure 6, the theoretical variation in water-depth with time is shown by the

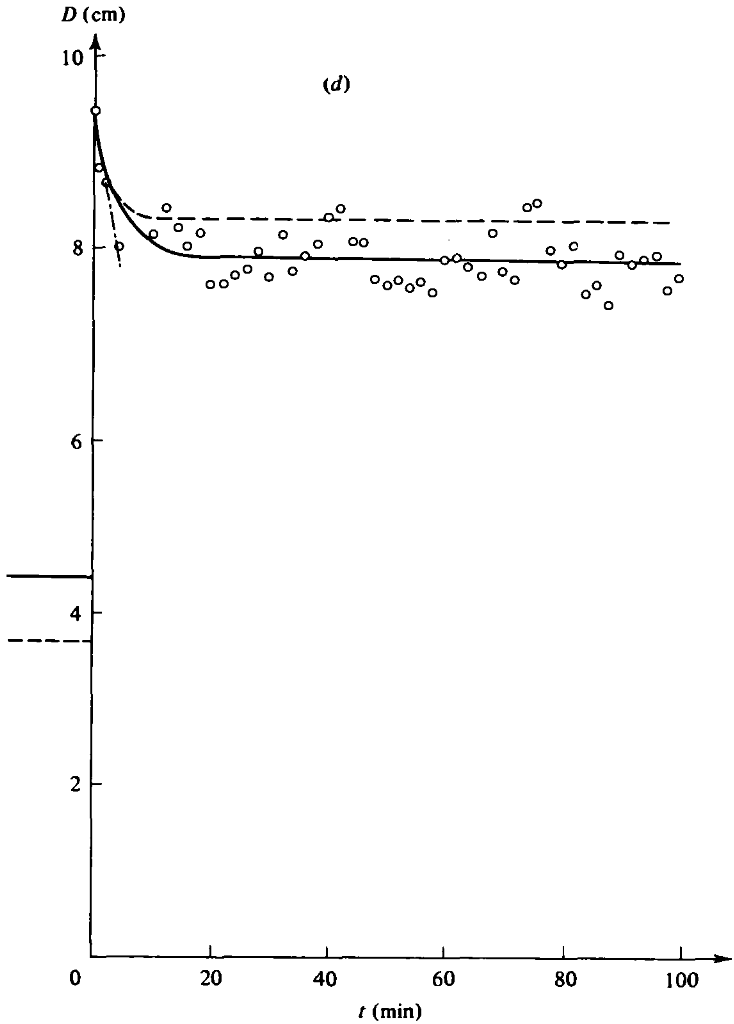

FIGURE 6 (d). For legend see page 447.

dotted lines. It is assumed that the initial changes in wavelength occur much more slowly than the changes in dune height, as argued by Fredsøe (1979). The initial change in water depth, just after the water discharge has been changed, is connected to the equilibrium depth by an exponential curve. The solid line is Gee's own best fit to his experimental results, depicted as circles. It is seen that the theory as well as the measurements yield the following general picture: at $t = 0$, the water depth changes suddenly because of the abrupt change in water depth and energy gradient. In the experiments where θ' is increased from a low value to a higher one (figure 6a, d and e) the depth is decreased due to a decrease in the dune height. At the lower values of θ', bed load is the dominant transport mechanism, while suspended load is significant at the higher values of θ'. This is the reverse of the picture described by Fredsøe (1979), (and verified by Gee's coarse sand experiment), where an increase in θ' results in an increase in dune height. However, in the coarse sand experiments the value of θ' was

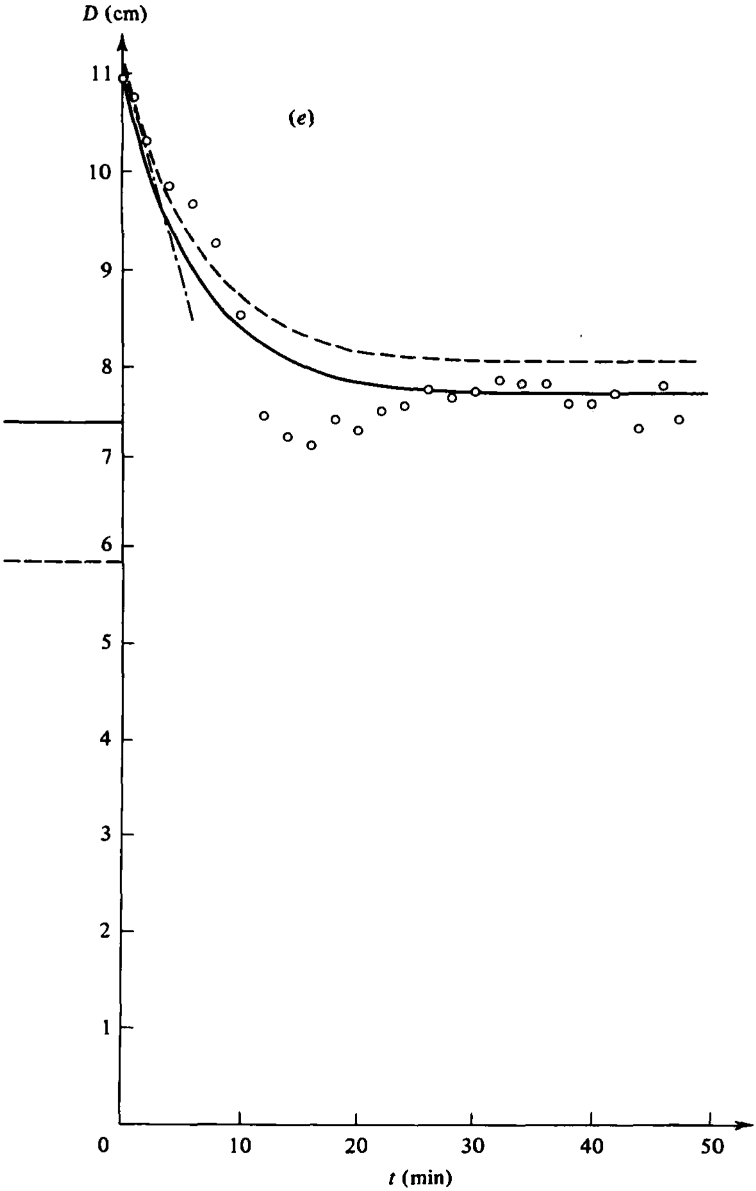

FIGURE 6. Changes in water depth with time after a sudden change in water depth. The circles are Gee's experimental data. ---, theory; ——, Gee's best-fit curve to experiments; -·-, initial change obtained from (9). Figure (*a*) describes 19R → 21, (*b*) 21 → 22, (*c*) 25 → 26, (*d*) 27 → 28 and (*e*) 33 → 34.

always so small that the amount of suspended load was negligible. If on the other hand θ' is reduced from a high to a low value (figure 6*b* and *c*) the depth increases as does the height of the dunes.

The quantitative agreement between theory and experiments is good concerning the initial change in water depth: the slope of the calculated curve does not deviate much from the measured one. The absolute predicted value of the water depth deviates within about 20 % of the observed value. These deviations are systematic: at low values of θ', the predicted depth is smaller than the measured one, while the opposite is the case at large values of θ'. This is related to two well-known phenomenon:

(i) at low values of θ', ripples and dunes exist together if the sediment is fine. Engelund & Hansen (1972) suggest that ripples exist if $U_f'd/\nu \lesssim 11{\cdot}6$, where ν is the kinematic viscosity of water (transition between hydraulic smooth and rough bed). Ripples increase the flow resistance and hence the water depth for specific water discharge and energy gradient.

(ii) at large values of θ' the concentration of suspended sediment close to the bed is large, which reduces the von Karman constant, see for instance Raudkivi (1976, pp. 187–189). This reduction involves a decrease in the water depth for given hydraulic conditions compared with clear water flows.

The development and destruction of ripples occur much faster than for dunes because of the small dimensions of ripples, so it does not influence the changes in water depth except for a very short time, probably of the order of that required for the flume to adjust to the new flow conditions, which is about 1–2 min.

From Gee's best-fit curve, he calculated the duration T of the transition defined as the time at which 95 % of the change in water depth has taken place. In table 1, this value can be compared with the theoretical one. The agreement between the two quantities is within a factor 2. It is seen that the time scale for changes is significantly smaller at high values of θ' than at low values.

Furthermore, the excursion C, defined by Allen (1976) as

$$\frac{1}{L}\int_0^{\tau} a(t)\,dt = C, \tag{39}$$

is given in table 1. τ is a time scale which, for reasons of comparison with Fredsøe (1979), has been put equal to the time it takes for the dune to change its height by half the difference between the new and the former equilibrium wave heights. C varies from 1·2 to 3·3, and there is no indication that C is a function of θ'. In the former paper by Fredsøe (1979), C varied between 2·5 and 5·8, so C is of the same order regardless of whether suspension is present or not, namely somewhat larger than unity.

6. Periodically varying water discharge in rivers

From (9), the initial change in dune height can be found if the bed shear is suddenly changed by a certain value. By linearization of this formulae, it is possible to obtain a step function for changes in dune height due to a sudden small change dQ in water discharge, as demonstrated by Fredsøe (1979, § 4). Hence, if the water discharge varies weakly with time according to

$$dQ = dQ_0 \sin \omega t, \tag{40}$$

where $\omega = 2\pi/T$ is the cyclic frequency and T the period, it is possible to calculate variations in dune height, water depth and effective bed shear stress (and hence sediment transport) by applying a convolution integral on the impulse-response function. In this way, the above-mentioned variations will be given by

$$\left.\begin{aligned} d(H/L) &= A_{H/L}\sin(\omega t - \Phi_{H/L}),\\ dD &= A_D \sin(\omega t - \Phi_D),\\ d\theta' &= A_{\theta'}\sin(\omega t - \Phi_{\theta'}). \end{aligned}\right\} \tag{41}$$

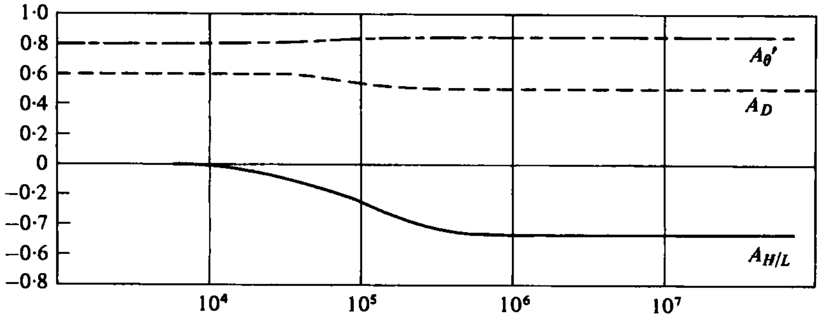

FIGURE 7. Variation in relative amplitudes with T for $Q = 1\ \mathrm{m^3\,s^{-1}}$, $I = 1{\cdot}5\,.\,10^{-4}$ and $d = 0{\cdot}20$ mm.

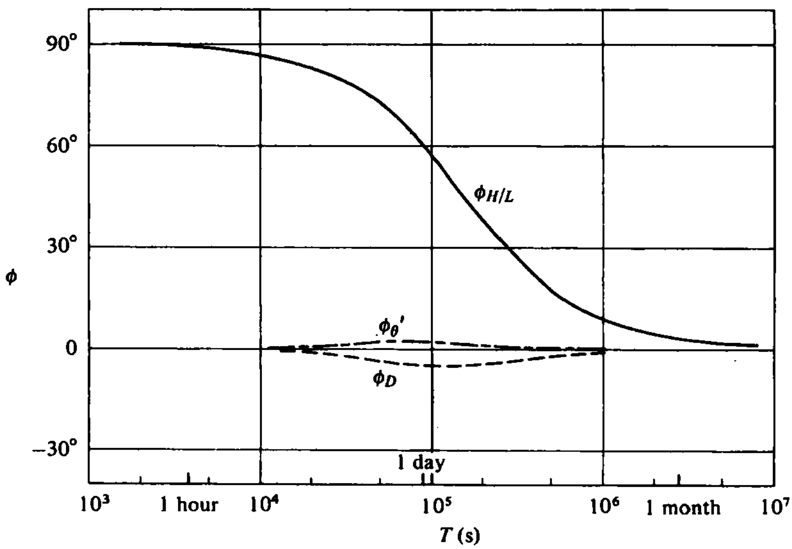

FIGURE 8. Variation in phases with T for the data in figure 7.

As the present calculations are quite similar to those presented by Fredsøe (1979, §§4 and 5), they are omitted here and only results of the present analyses are given.

In the analysis, the equilibrium states for dune height, water depth and bed shear stress were calculated by use of the resistance formulae

$$\theta' = 0{\cdot}06 + 0{\cdot}4\,\theta^2 \tag{42}$$

(Engelund & Hansen 1972). This formulae is valid as long as the bed is dune covered except in the region where transition to plane bed occurs. However, (42) is able to predict the decrease in the dune height at high values of θ as demonstrated by Fredsøe (1979), who predicted that the ratio H/D has a maximum of $\theta = 0{\cdot}35$. If θ is further increased, H/D will decrease owing to the effect of suspended sediment.

Figures 7 and 8 show an example of the variation in the phases and amplitudes defined by (41) for flow over a bed not far away from transition to a plane bed. The hydraulic data are in fact the same as those used in figures 5 and 6 in the paper by

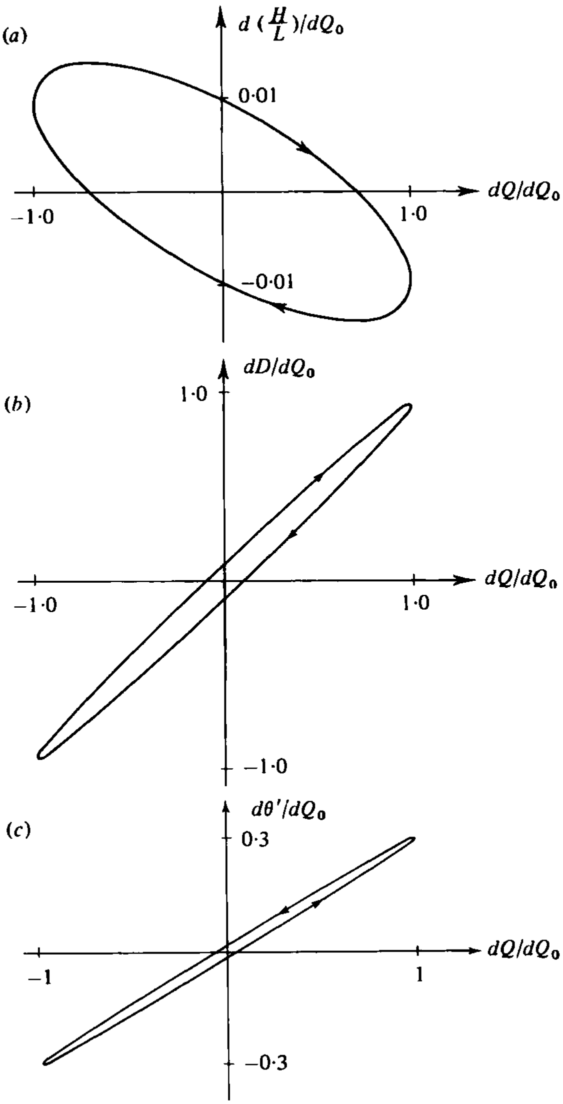

FIGURE 9. Variation in H/L, D and θ' with water discharge for $T = 1{\cdot}5 \times 10^6$ s for the data in figure 7.

Fredsøe (1979), but the mean grain diameter is reduced from 0·60 to 0·20 mm, so θ is changed from 0·30 to 0·64. In this last case, the ratio between suspended sediment load and bed load is about 3, while in the former case it is very close to zero.

Like the flow over coarse sand (0·60 mm), it is now seen from figures 7 and 8 that the variations in H/L still lag behind the variations in dQ with a phase difference varying from 90° for rapid changes to 0° for very slow changes. However, the amplitude $A_{H/L}$ now becomes negative because H/L decreases as the water discharge increases. Further, it is noticed that $\Phi_{H/L} = 45°$ for $T \sim 1{\cdot}5 \times 10^6$ s, which is less than two days. In this time, the dune has travelled its own length 3 times. Hence, the change in dune dimensions occurs much faster in the case of fine sand than the case of coarse sand for specific hydraulic parameters: if the mean diameter of the sand was instead 0·60 mm, $\Phi_{H/L} = 45°$ for T equal to 12 days, as calculated by Fredsøe (1979).

The present river model is based on the assumption that longitudinal variations in specific water discharge and in water depth can be neglected. However, the results given above indicate that the changes in dune dimensions at large values of θ' occur so rapidly that, for instance, calculations of flood waves in alluvial streams must take into account variations in the friction factor because of changes in dune dimensions.

In figure 9, the variation in H/L, D and θ' with Q is depicted for $T = 1{\cdot}5 \times 10^6$ s. It is seen that the loop in the relation between H/L and Q now circulates reverse compared with that depicted in Fredsøe (1979, figure 7), in which example only bed load was present. Loops in the ratio H/L of the same form as sketched in figure 9*a* are for instance depicted by Allen (1976, figure 3), while loops in depth similar to figure 9*b* are obtained in laboratory experiments by Simons & Richardson (1962, figure 2). However, no quantitative comparison is possible, as the water discharge varies over a wide range in both references.

7. Unsteady flow caused by temperature variations

As mentioned in the introduction, transition from dunes to a plane bed has been studied by stability analysis, by which it can be predicted that bed-load causes instability, while suspended load acts as a stabilizing factor. This result explains the transition to a plane bed because of a drop in water temperature with unchanged hydraulic conditions: because of the increased kinematic viscosity of water, the fall velocity of suspended sediment decreases so that the number of suspended particles increase, while the rate of bed-load transport remains essentially unchanged. This may lead to transition to a plane bed as depicted in Engelund & Fredsøe (1974, figure 5·7), which shows the results of the stability theory for the Missouri River, in which it has been observed that the dunes disappear during two months in the autumn 1966 without any significant change in the water discharge.

The time scale for this change is easily obtained from (9): if the bed shear stress is assumed to remain unchanged after a sudden change in water temperature, (9) yields

$$\frac{dh}{dt} = \frac{1}{(1-n)}\frac{d}{dx}(q_{s1}-q_{s2}), \tag{43}$$

because the rate of bed-load depends only on the bed shear stress. q_{s1} is the suspended load at temperature t_2, while q_{s1} is the original load at temperature t_1.

Equation (43) yields that the dune height will decrease as the temperature decreases. As a specific example, (43) predicts that the initial decrease in dune height will be equal 1·76 H/L meter per hour for the Missouri River data if the temperature is suddenly decreased 10° Celcius. The typical Missouri data used by Engelund & Fredsøe (1974) are: depth = 3·2 m, mean grain diameter = 0·21 mm and mean velocity = 1·50 m/s.

As mentioned before, Engelunds similarity theory breaks down in the transition region very close to plane bed. Further on, temperature effects are not included in the theory, so very little is known about the dune height and length in this region.

Combining (11), (35) and (36) suggest the following general formulae for dune height far away from the critical Froude number:

$$\frac{H}{D}\bigg/\left(1-\frac{H}{2D}\right) = q_b\bigg/2\theta'\left(\frac{dq_b}{d\theta'}+\frac{1}{1+\lambda}\frac{dq_s}{d\theta'}\right), \tag{44}$$

which predicts a decrease in the equilibrium dune height equal to 9 cm, from 15 to 6 cm as the temperature decreases 10° Celcius in the above-mentioned example.

While it is not possible to calculate L in this region, the constant C defined in (39) can be calculated, because it is independent of L. This is because the time scale τ is proportional to the dune length. For the specific example considered above, C is found to be 0·40 or smaller than the values obtained for changes in dune dimensions due to a sudden change in bed shear stress. This is because the bed load in case of temperature variations does not counteract the influence from the suspended sediment in the same active way.

8. Conclusion

The initial change in dune height after a sudden change in water discharge has been investigated at high sediment transport rates, where the rate of suspended load is significant. The transport of sediment is split into transport of bed-load and transport of suspended load, which is essential for the analysis, because only the bed-load transported over the dune crest will settle on the dune front and hence contribute to the migration velocity of the dunes.

The analysis predicts that if the bed shear stress is increased, the bed-load will try to increase the dune height, while the suspended load acts against this, trying to destroy the dune. Hence, if large amounts of suspended sediment are present, the dune height will decrease as the bed shear increases. In a river with weakly varying water discharge, this implies that the bed shear stress is smaller in the rising state than in the decreasing state, which is the reverse of the conditions at small sediment transport rates, where bed-load is the dominating transport form. If the temperature varies, the theory predicts that the dunes respond rather fast to these variations by reducing their height as the temperature decreases and vice versa.

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